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A model to evaluate coupled driving-and-charging incentives for electric vehicles

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Abstract
As Electric Vehicles penetration increases, more impacts on urban systems are observed and related to both driving (e.g., on traffic congestion and reduced pollution) and charging (e.g., on the electrical grid). Therefore, there is a need to design coupled incentive mechanisms. To propose and numerically to evaluate such incentives, a game theory model is adopted. Its originality comes from the coupling between the charging cost and the driving decisions: to drive downtown or to charge at a e-Park & Ride hub with solar panels and then take public transport, in order to reach destination. Optimal ticket fares and panel surfaces are computed using real photovoltaic production data.
Keywords: electric vehicles, dynamic charging, mobility concept, optimization

1 Introduction

1.1 Motivation

At a local scale, urban well-being is sensitive to road usage and its impact on traffic congestion, local air pollution and noise. Electric Vehicles (EV), considering both battery and plug-in hybrid technologies, is a promising solution to these issues. However, the forecasted high penetration of EV (see middle scenario in [1]) may lead to local grid constraints, e.g., transformers aging and power losses. Even if the penetration rate of EV is not yet really significant at the national scale, it can already be substantial at a local scale[1]. This "grid congestion" problem has to be considered as a key factor for the large scale deployment of EV. Therefore, a model to evaluate coupled driving-and-charging incentives for EV can be very useful to understand and predict future performances of such complex interaction between transport and energy. The *flexibility* of EV charging - in terms of compatibility with end users mobility needs and technical capabilities for load management - makes it a significant tool in "Demand Response" mechanisms [2] which is an emerging field in "Smart Grids". Such scheduling techniques consist in shifting/adapting the consumption profile by, e.g., postponing usages in time, or reducing the level of power consumed, with different objectives for the electrical system: local management of production-consumption balance, mitigating the impact on the electrical grid [3], etc. This is totally innovative compared to the traditional paradigm of the electrical system, where almost only generation units were flexible to ensure its effective operation[2]. In this context, taking into account charging strategies into everyday EV driving decisions will become an important issue in smart cities, particularly for urban

[1]See e.g. the case of "Île-de-France", with more than 20 000 EV in circulation: `http://www.automobile-propre.com/dossiers/voitures-electriques/chiffres-vente-immatriculations-france/` (in French).
[2]For other tasks than EV charging (heating, cooking, etc.), there is less potential to "smartly" schedule the associated electricity consumption profile. Currently, the main flexibility in France is water-heating, controlled through on/off peak fares.

networks [4]. Another important problem is the design of charging incentives (e.g., under the form of pricing or services) to share – in space and time – public EV Charging Stations (or EVCS) [5].
To solve these problems, EV driving decisions must be also taken into account. This coupling is clearly observable during widespread holidays journeys or particular events: the majority of driving EV need to charge at public EVCS, where there could be a significant waiting time and available power reduction (when allocated/shared between plugged EV) due to simultaneous power demands. As an example of incentive mechanism, Tesla EVCS proposes a differentiated service and adapt the charging prices in order to encourage EV to charge in empty EVCS rather than congested ones[3]. Another example from the French company CNR (*Compagnie Nationale du Rhône*) is the "Move In Pure" charging subscription: in order to guarantee an EV charging green power sourcing, drivers are incited to charge at specific hours of the day (resp. locations) when (resp. where) renewable energy is available. In a more futuristic vision, the EV charging-and-driving coupling can be transposed into a charging-by-driving one, with an inductive charging system (under the road) as suggested in [6]. Finally, Park & Ride hubs – associated with public transport – are in vogue to mitigate congestion and local pollution in urban areas: up to 18 000 parking spaces are expected at Paris gates by 2021[4]. This alternative multimodal solution represents a great opportunity for smart charging. The model presented in this work takes into account this coupled framework between EV driving and charging decisions in order to offer an accurate representation of EV behavior. A direct application of the proposed model allows testing incentives aimed at, e.g., mitigating the impact of EV charging on the electrical grid, minimizing the proportion of gasoline vehicles into city center or maximizing the profit of Charge Point Operators (CPO). Having this context in mind, we propose a scenario in which a population of electric and gasoline vehicles follow the same journey from a sub-urban area to a city center, which corresponds to regular commuting patterns.

1.2 Related methodologies

Basic Traffic Assignment Problem (TAP) with single-class drivers (meaning that there is only one type of vehicle) is defined and studied in [7]. Under certain conditions (drivers equally affected by traffic congestion and increasing cost functions), it is shown that there is a unique solution[5] to this problem. In recent years, there has been an increasing interest for mixed TAP where two or more classes of vehicles are considered [8] (e.g., electric and gasoline vehicles). The uniqueness of the solution in mixed TAP is proved in [9] when the cost functions are the same for every driver, up to an additive constant.
On the charging side of the problem, the water-filling schedule of [10] will be used. The coupling of the driving and charging problems is studied in particular in [11] and [12]. However, [11] focuses only on a single class of vehicles and [12] considers that the EV charging need is constant and does not depend on their driving decisions.

2 E-Park & Ride hub scenario

Note that the scenario considered here is one of the many practical applications of the generic model developed in our previous work [13]. This work focuses typically on daily commuters who want to get to their workplace in the morning: they come from the suburb area (Origin O in Fig.1) and head the city center (Destination D). This city is concerned with traffic congestion and local pollution, so an e-Park & Ride hub is built on the outskirts of the city to limit the number of vehicles downtown. In this scenario, when commuters arrive at the hub, they can choose between two transport modes. First, they can park at the hub and finish their trip by public transport ($publ$ in Fig. 1). Second, they can drive past the hub into the city center with their private vehicle ($priv$). At the hub, a Charge Point Operator (CPO) is in charge of smart public EVCS and solar panels. The CPO is separate from the various network operators (of the electrical grid and of the traffic network). By the way, the relationships between CPO and those network operators are not considered in our framework here. Indeed, our work is focused on EV drivers decisions and their impact. Note that these higher level economic relationships between CPO and network operators can be considered on top of our model, this is clearly a perspective of the work.
Note that while the private transport mode may be faster, the public one may be cheaper thanks to incentives, like public transport ticket fare discount or cheaper charging service for EV due to a local

[3]`https://www.tesla.com/support/supercharging`.
[4]`www.iledefrance-mobilites.fr/actualites/18-000-places-de-parc-relais-2018/`.
[5]The solution concept, explained in next Section, is close to the Nash equilibrium concept in game theory.

electricity production at the hub. The aim of this work is to model and then to predict the choice of commuters and to find how it may be affected by various incentives like the two mentioned above.

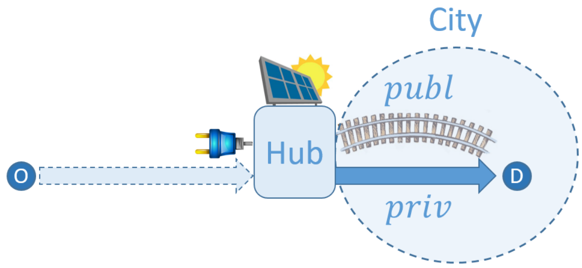


Figure 1: Schematic representation of the charging hub scenario: commuters can either choose to leave their vehicle at the hub and take public transport ($publ$), or drive all the way to their destination ($priv$). A local source of renewable energy is available at the hub.

2.1 Route choice

2.1.1 Model assumptions

- Two types of vehicle are considered: an electric one (denoted EV and associated with subscript e) and a gasoline one (GV, associated with subscript g). Each commuter is associated with one of the two vehicles and the proportion of EV among them is denoted by X_e in the model[6]. The proportion of GV is then given by $X_g = 1 - X_e$. The choice made by all commuters between the two transport modes of Fig. 1 is represented by the two variables $x_{e,publ}$ and $x_{g,publ}$, which are respectively the proportions of EV and GV choosing the public transport mode. Note that the proportions of vehicles of type $s = e, g$ choosing the private transport mode may be easily deduced: $x_{s,priv} = 1 - x_{s,publ}$.

- The decision process of commuters is assumed *rational*, meaning that they choose the transport mode (*publ* or *priv*) with minimal cost. Here, the costs considered are travel duration (by private or public transport), energy consumption (electricity for EV and fuel for GV) and the ticket fare (for public transport only).

2.1.2 Costs functions

The first type of costs considered is related to travel duration and/or delay from the hub to the destination, which are perceived equivalently by EV and GV:

a) Travel duration costs

– For the private mode, it depends on the total proportion[7] of vehicles driving downtown $x_{priv} = x_{e,priv}X_e + x_{g,priv}X_g = (1 - x_{e,publ})X_e + (1 - x_{g,publ})X_g$ due to congestion effects [14] and is expressed as:

$$\tau_{priv} \times \underbrace{\frac{l}{v}\left(1 + 2\left(\frac{x_{priv}}{C}\right)^4\right)}_{d_{priv}(x_{priv})}, \tag{1}$$

where the function $d_{priv}(.)$ is the estimated travel duration on the road downtown: the higher the flow x_{priv}, the higher the travel duration and $x_{priv} = 0$ yields the (minimal) "free-flow" travel time. The parameters of the problem are set as follows, unless otherwise specified:

- $\tau_{priv} = 10$€/h value of time when driving, based on a French government report[8],

[6] In numerical tests, $X_e = 50\%$ which is in line with 2035 predictions for France (see middle scenario of [1]).

[7] Here, proportion and number of vehicles are equivalent, as the total number of vehicles is fixed.

[8] `http://www.strategie.gouv.fr/sites/strategie.gouv.fr/files/archives/Valeur-du-temps.pdf`.

- $l = 5$km length of the road, approximately the radius of Paris,
- $v = 50$km/h speed limit, as in French urban areas,
- $C = 1$ capacity of the road, expressed in proportion of the total number of vehicles as x_{priv}.

Note that the factor 2 in (1) yields that if all vehicles choose to drive downtown ($x_{priv} = 1$), the corresponding travel duration is multiplied by 3 compared to the empty road situation (or free-flow) due to traffic jams (see Figure 2)[9].

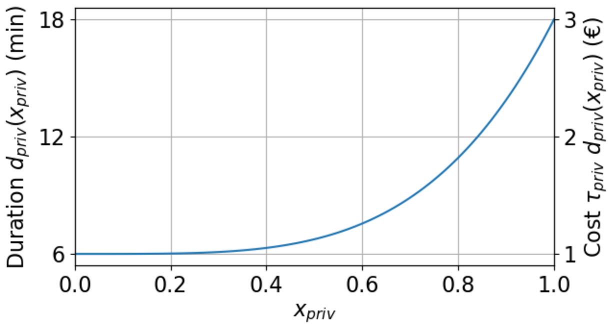


Figure 2: Travel duration $d_{priv}(.)$ (and the associated cost $\tau_{priv} d_{priv}(.)$, along the right axis) for vehicles driving downtown depending on their number x_{priv}. *If all vehicles choose the private transport mode, the associated duration will be three times higher than the free-flow case (when all vehicles choose public mode).*

– For the public transport mode linking the hub and the destination, the travel cost is assumed constant:

$$\tau_{publ} \times d_{publ} \, , \tag{2}$$

- $\tau_{publ} = 12$€/h value of time in public transport[8], which is perceived by commuters as less comfortable than personal vehicles,
- $d_{publ} = \frac{l}{v} = 6$ min constant travel time of public transport, which was chosen equal to the free flow travel time of the private mode. Indeed, there exist reserved pathways for public transportation in several cities like Paris, so that congestion can be considered as marginal.

The duration cost of the public mode is then equal to the fixed value $\tau_{publ}\ d_{publ} = 1.2$€ and is higher than the free flow cost of the private mode. this induces trade-off decisions between both strategies for vehicles.

b) Energy consumption cost. It corresponds to the energy consumed by the vehicle from the origin to the destination; it is different for EV and GV. The expression of this cost for vehicles of type $s = e, g$ which have chosen transport mode $r = publ, priv$ is as follows:

$$l_r \times m_s \times \lambda_s \, . \tag{3}$$

Note that here, the consumption model is assumed to be distance-dependent.

- l_r is the total distance driven by the vehicles which have chosen transport mode r, and is equal to:
  - $l_{publ} = 10$km distance between the origin and the hub, so that the two-way trip between origin and destination is 30km, the daily average individual driving distance in France (following ENTD[10] 2008),
  - $l_{priv} = l_{publ} + l = 15$km,

[9] A typical value for inter-urban areas [15].

[10] *Enquête Nationale Transports et Déplacements*: `https://utp.fr/system/files/Publications/UTP_NoteInfo1103_Enseignements_ENTD2008.pdf` (in French).

- m_s is the electricity or fuel consumed per distance unit and is supposed constant (e.g., it does not depend on speed profiles):
  - $m_e = 0.2$kWh/km, following [16],
  - $m_g = 0.06$L/km (Liter/km),
- λ_s is the charging/fueling unit price:
  - For EV, the key distinction made here is that it depends on the transport mode chosen.
    Public mode: At the hub, this charging unit price λ_e will depend on the total charging need $L_e(x_{e,publ})$, proportional to the number of EV parked in the hub: for example if there are few EV at the hub ($x_{e,publ}$ close to 0), there is enough free electricity[11] produced at the hub to provide the charging need of these EV. This price is obtained by solving a charging problem, which is detailed in next Section.
    Private mode: Downtown, there is a standard constant electricity fare $\lambda_e^0 = 20$c€/kWh, which corresponds to the electricity unit price in France (15c€/kWh) with an additional cost (5c€/kWh) meant for the charging operation[12].
  - $\lambda_g = 1.50$€/L is considered constant.

c) Public transport ticket fare. It is the same for EV and GV: $t_{publ} = 2$ €, which corresponds approximately to Paris (single ticket) fare.
Finally, the total costs for each type $s = e, g$ of vehicle which have chosen transport mode $r = publ, priv$ are given in the following table (where $\mathbf{x} = (x_{e,publ}, x_{g,publ})$):

Total costs	Public transport mode	Private transport mode
EV	$c_{e,publ}\,(x_{e,publ}) = \tau_{publ}\, d_{publ} + t_{publ} + l_{publ}\, m_e \lambda_e\,(L_e(x_{e,publ}))$	$c_{e,priv}\,(\mathbf{x}) = \tau_{priv}\, d_{priv}(x_{priv}) + l_{priv}\, m_e \lambda_e^0$
GV	$c_{g,publ} = \tau_{publ}\, d_{publ} + t_{publ} + l_{publ}\, m_g \lambda_g$	$c_{g,priv}\,(\mathbf{x}) = \tau_{priv}\, d_{priv}(x_{priv}) + l_{priv}\, m_g \lambda_g$

Note that the driving and charging operations are coupled: the mode choice depends on the charging cost (charging impacts driving) while the EV charging need depends on their driving consumption (driving impacts charging).
According to the rationality assumption, each commuter chooses the transport mode with minimal total cost, under *complete information*: he knows all the total cost expressions presented in previous table and know that all the other commuters want to minimize their total cost too. By all acting rationally in this sense, commuters will reach a certain distribution of choices between the public and the private modes. Such a distribution is denoted by $\mathbf{x}^* = \left(x_{e,publ}^*, x_{g,publ}^*\right)$ and is called a Wardrop Equilibrium (or WE in game theory literature) [17]. This equilibrium situation gives a model of commuters behavior in a stable regime where no commuter has an interest to change his choice unilaterally. This is a typical situation, after some learning periods, when drivers determine their route or follow a guidance app, for their everyday journey form their home place to their job place. The proposed approach can thus be used to evaluate various incentive mechanisms numerically - in a planning stage or tool - in order to "select" a particular equilibrium before it will be observed in practice[13], as done in Section 3.
Before that, next Section introduces the hub charging operation and the determination of the charging unit price in more details.

2.2 Hub charging operation

This section explains how the charging unit price λ_e at the hub is determined optimally and depends on the proportion of EV choosing the public transport mode, and thus charging at the hub.

[11] The PV marginal production cost is supposed to be zero.
[12] This fare is comparable to the 25c€/kWh proposed by Tesla for fast charging - necessarily a little more expensive; see aforementioned webpage.
[13] Observe that this concept is now commonly used in many operational (public) transportation planning tools for the "route choice" step in four-steps models.

2.2.1 Charging scenario

When commuters arrive at the hub, those having an EV leave it plugged in during work hours and let the CPO chooses the charging schedule ("centralized optimization problem"). For example, the CPO might refer to a state entity which built a smart charging infrastructure in order to minimize social costs, or to a private company opening its parking lot to the public. The CPO determines the individual charging profiles of all EVs connected at the hub during the day. Here, instead of solving this optimal scheduling problem with the per-EV profiles - which is a topic in itself, see e.g. [18], an aggregate version of this problem is tackled. It consists in considering an optimization problem in which the variable is the aggregated charging profile, i.e. the sum of the individual ones. With a significantly lower complexity of resolution (an explicit solution is available), it provides a good approximation of the aggregate charging cost, from which the charging unit price is derived. On top of that, the hub owns a local source of energy like photovoltaic (or PV) panels. Therefore, performing most of the charging operation around noon when PV panels are at their production peak may be a better solution for the CPO rather than a uniform charging profile. At the end of the day, the total cost/impact of the charging operation affects the hub charging unit price such that the CPO is at a break even point.

2.2.2 Modeling of charging problem

Aggregated charging need: It is assumed that before leaving the suburb areas (corresponding to the Origin on Figure 1), all EVs' state of charge is full, so that their charging need corresponds exactly to the electricity consumed during their trip from their origin to the hub. This assumption may be lifted by grouping the vehicles with the same initial state of charge. As the consumption model is distance-dependent, the aggregated charging need $L_e(x_{e,publ})$ for all EVs is then proportional to the total travelled distance $l_{publ}\ x_{e,publ}$ by the EV choosing the public transport mode:

$$L_e(x_{e,publ}) = l_{publ}\ m_e\ x_{e,publ}\,. \tag{4}$$

The CPO commits to fully charge all EV at the hub, i.e. the whole aggregated charging need L_e.

Temporal charging scheduling: The CPO determines which portion of the total charging need L_e that has to be charged during each working hour of the day in order to minimize total charging costs. Note that these costs are supposed to be aligned with the costs/impact of the charging operation on the electrical grid (introduced later in this Section); in practice, a specific electricity contract would be signed between the CPO and the grid operator, determining the remuneration of the CPO for such "effort"[14]. To simplify, the charging operation is assumed to take place only between 9 a.m. and 5 p.m., when all the EV which have chosen the public transport mode are likely to be plugged in (considering a hub with enough capacity). EV arriving at the hub before 9 a.m. or leaving after 5 p.m. will not be charged outside this period, so that the scheduling in this work might not be exactly optimal and the resulting charging unit price might be overestimated. The charging period consists in eight time slots of one hour eachand the CPO decides the load $\ell_{e,t}$ to charge during each time slot $t \in \{1, \dots, 8\}$, so that the aggregated charging need is satisfied:

$$\sum_{t=1}^{8} \ell_{e,t} = L_e(x_{e,publ}) \quad \text{(in kWh)}\,. \tag{5}$$

Photovoltaic production: The CPO determines the aggregated charging profile taking into account its local PV energy production (assuming the PV production of the day is known when solving the charging problem, typically "just before" 9 a.m.). For each time slot $t \in \{1, \dots, 8\}$, the PV energy produced at the hub is denoted by $p_t \geq 0$. Figure 3 shows the open source data[15] from [19] considered for the PV production. This figure shows the energy produced each working hour (averaged over the year 2014) per squared meter of a photovoltaic panel with a nominal power of 360W and a surface of 2.06m^2 located in Paris[16].

[14] In France see the "Offres de Raccordement Intelligentes" by Enedis for an example of such remuneration scheme.

[15] Available at `https://www.renewables.ninja/`.

[16] Example taken from `https://us.sunpower.com/solar-resources/products/datasheets/`.

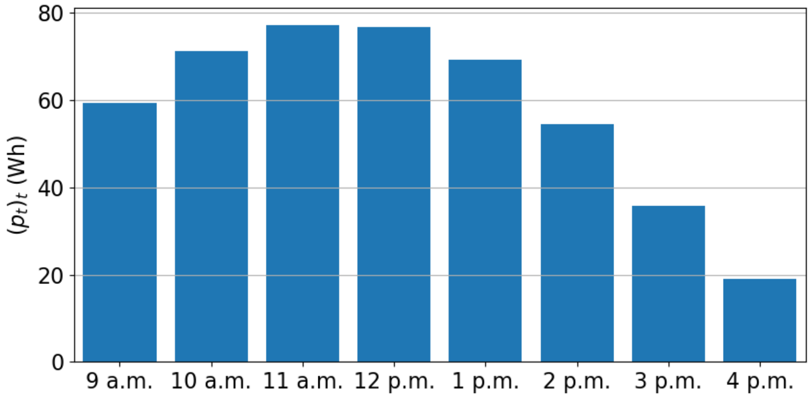


Figure 3: Energy produced by 1m^2 of a solar panel in Paris during working hours (averaged over the year 2014).

Cost/impact on the local electrical grid: The cost/impact of the charging operation on the local electrical (distribution) grid at time slot $t \in \{1, \ldots, 8\}$ depends on the net load ("seen" from the grid) $\ell_t = -p_t + \ell_{e,t}$; it writes:

$$f(\ell_t) = \begin{cases} 0 & \text{if } \ell_t \leq 0\,, \\ \eta \ell_t^2 & \text{if } \ell_t > 0\,. \end{cases} \quad (\text{in } €) \tag{6}$$

This cost is typically quadratic when the net load is positive and zero if not (see Figure 4). This form of function means that: when the charging operation uses only PV production, there are no grid costs: when the CPO needs electricity from the grid (i.e., $\ell_{e,t} > p_t$), costs are considered under an increasing and convex form standardly used in optimization/game-theory smart grid models to represent local grid congestion effects [10]. Following are a few observations regarding this grid cost modeling: 1. Regarding the particular choice of a quadratic function, note that the following study is still valid with more general monomials (cost function f with a higher degree); 2. Because the cost function in (6) does not depend on the variables at the other time slots (in particular on the previous net load ℓ_{t-1}), this "proxy" does not include dynamical (e.g., transformer temperature inertia as in [3]) nor locational effects; 3. A local electricity storage for PV production is not considered here; its presence could decrease the net load and, in turn, the impact on the grid of (6); 4. Finally, this cost function incentivises the CPO to maximize the self-consumption of its PV production; this fact will be detailed further.

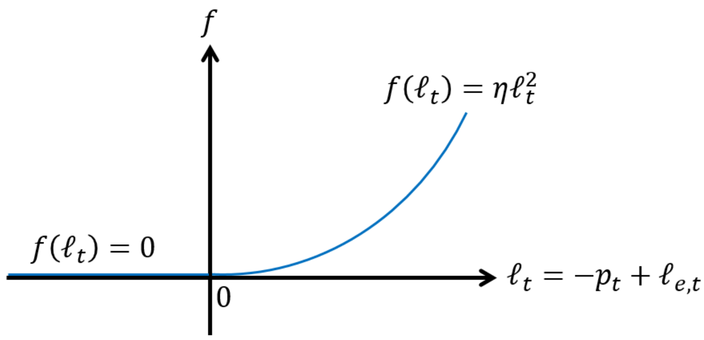


Figure 4: Cost/impact on the electrical grid of EV charging and PV production (through the net load ℓ_t) at time slot t. *If PV production is higher (resp. smaller) than EV charging load, there is no (resp. a quadratic) impact.*

Charging problem and solution: Formally, the charging problem solved by the CPO writes:

$$\min_{(\ell_{e,t})_t} \sum_{t=1}^{8} f(-p_t + \ell_{e,t})\,, \quad \text{s.t.} \quad \begin{cases} \forall t\,, \;\; \ell_{e,t} \geq 0\,, \\ \sum_{t=1}^{8} \ell_{e,t} = L_e(x_{e,publ})\,. \end{cases} \tag{7}$$

The solution of this problem only depends on the total PV energy produced during working hours $E = \sum_{t=1}^{8} p_t$ (relatively to L_e) and not on the profile $(p_t)_t$ shape. The PV profile considered corresponds to a PV panel surface of 125m^2 (equivalent to the area of approximately 87 parking spots), with a mean (average over working days) total production of $E = 57.9$kWh during working hours per day.

- If the aggregated charging need $L_e(x_{e,publ})$ verifies $L_e < E$, any charging profile below the PV production is optimal, since the associated cost is zero.
- If $L_e = E$ (which corresponds to the charging need of 29 EVs), the optimal scheduling has to perfectly match the production.
- If $L_e > E$, all PV production is consumed and the remaining charging need has to be equally shared between all time slots such that the net load taken from the grid be constant.

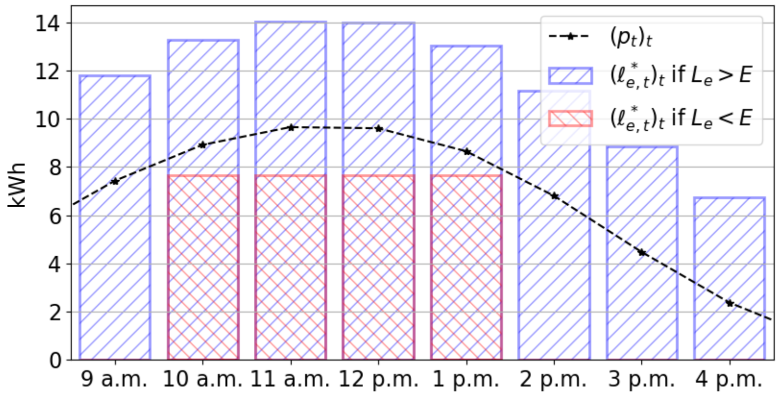


Figure 5: Water-filling optimal scheduling of the charging operation with 125m^2 of solar panel (in black), for 15 EV (in red) and 45 EV (in blue). *For 15 EV, any scheduling using only PV production is optimal, while for 45 EV, the only optimal scheduling uses the whole PV production plus the same amount from the electrical grid at each time slot.*

The optimal charging schedule solution of (7) gives a minimal total cost/impact on the electrical grid denoted by $C(L_e)$ (in €), which is equal to[17]:

$$C(L_e) = \begin{cases} 0 & \text{if } L_e \leq E\,, \\ \frac{\eta}{8}\,(-E + L_e)^2 & \text{if } L_e > E\,. \end{cases} \tag{8}$$

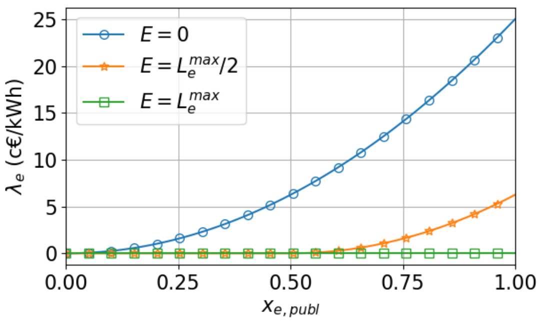
Having optimally scheduled the aggregated charging need $L_e(x_{e,publ})$ in different time slots, the CPO then determines the charging unit price (for $L_e > 0$) as follows:

$$\lambda_e(L_e) = \frac{C(L_e)}{L_e} \quad (\text{in €/kWh})\,. \tag{9}$$

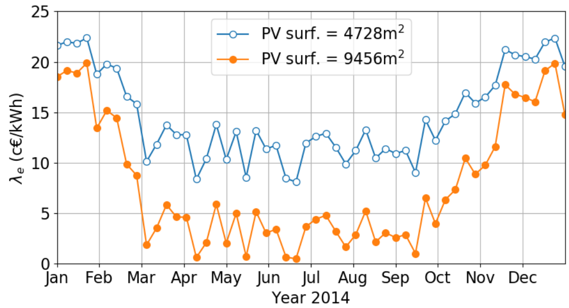
This way, the CPO makes EV pay equally (per energy unit) for the total charging cost caused by their aggregated electricity consumption need. Note the threshold role played by the total PV production E during working hours: EV start paying for the charging operation only if E is not sufficient to provide for the total charging need $L_e(x_{e,publ})$; otherwise the charging operation is free.
The parameter $\eta = 2$€/kWh^2 was adjusted so that the maximal charging unit price λ_e at the hub, which occurs when there is no PV production ($E = 0$) and all EVs choose public transport - and thus charge at the hub ($L_e^{\max} = l_0 m_e X_e$), is equal to $\frac{5}{4}\lambda_e^0 = 25$c€/kWh (see the circle markers in Figure 6). Note that this maximal price corresponds to the Tesla charging unit price in France[3] and is higher than the fixed price downtown λ_e^0.

[17]This minimal total cost C corresponds to the *value function* concept in optimization.

Figure 6 shows the impact on the charging unit price of the proportion of EV choosing the public transport mode (Figure 6a) and of the daily PV production E (Figure 6b). The total number of vehicles is 1 000 (500 EV). Then, for Figure 6a, $E = L_e^{\max}/2$ corresponds to the daily PV production of a 9456m^2 surface (averaged over year 2014). In this case, the charging service is free when less than 60% of EV choose to park at the hub, and the maximal charging unit price is 6c€/kWh (still relatively cheap). Observe that this shows that the charging unit price depends on the driving choices made by EV, so that the driving and charging operations are coupled.
Figure 6b shows the charging unit price at the hub if all EV choose the public transport mode, as a function of PV production throughout year 2014. For both PV panel surfaces, while during winter this price almost reaches the maximal price (25c€/kWh), it decreases dramatically during summer, when it could be more profitable to have an installed PV surface of 4 728m^2.



(a) As a **function of the number of EV** choosing public transport mode, with different PV production E.



(b) As a **function of PV production**, for two different PV surfaces, with all EV at the hub ($x_{e,publ} = 1$).

Figure 6: Charging unit price (in c€/kWh) at the hub. *The left (resp. right) figure highlights the influence of EV charging demand at the hub (resp. PV sizing and seasonal effects).*

3 Numerical experiments

3.1 Wardrop equilibrium representation

When the charging unit price at the hub λ_e introduced in previous Section is an increasing function of the total charging need $L_e(x_{e,publ})$ (which is the case here), a unique Wardrop Equilibrium (WE) exists - please refer to our previous work [13], Corollary 1. This equilibrium corresponds to a situation where no vehicle could lower its cost by choosing the other transport mode. To illustrate the concept of WE, we consider the parameters values set in Section 2.1.2 and no PV production (see the blue curve in Figure 6). The WE corresponding to this particular case is $(x^*_{e,publ}, x^*_{g,publ}) = (0.11, 0)$, meaning that no GV choose the public transport mode while few EV do so. To understand why, the different EV costs are shown in Figure 7 for any proportion $x_{e,publ}$ of EV choosing the public mode, with fixed $x_{g,publ} = x^*_{g,publ}$ (at its WE value). For example, if there were no EV choosing the public transport mode ($x_{e,publ} = 0$, on the extreme left of Figure 7), the total cost for EV choosing the private mode would be higher than for those choosing the public one, due to the congestion effect on the travel duration. Thus, $x_{e,publ} = 0$ cannot be an equilibrium situation, as some EV would prefer the public mode which is cheaper. Similarly, too many EV at the hub ($x_{e,publ} > 0.11$) would lead to $c_{e,publ} > c_{e,priv}$, so that it is not a WE as some EV would rather choose the private transport mode. In turn, Figure 7 shows that there is a unique WE $x_{e,publ} = 0.11$. Note that in this case total costs are equal between the two transport modes, so that no EV would rather choose the other mode. Also note that the monetary cost for vehicles at the hub (blue dotted line) are made of a fixed part (the ticket fare), and of a variable part (the charging cost) which depends on the proportion $x_{e,publ}$ of EV choosing the public transport mode.

3.2 Sensitivity to ticket fare

Thanks to the WE obtained in the proposed model, network operators are able to predict the number of EV and GV choosing the public or the private transport modes, whatever the problem parameters may be.

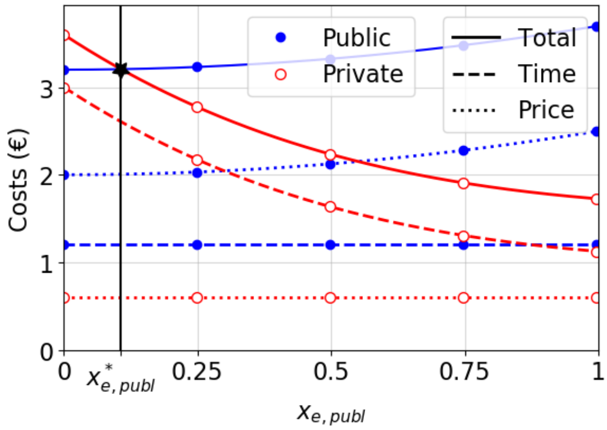


Figure 7: EV costs as a function of the proportion $x_{e,publ}$ of EV choosing public transport. In blue (resp. red) is the cost for EV choosing public (resp. private) transport mode. The dotted lines refer to the monetary costs (consumption and ticket fare for public mode; only consumption for private mode) and the dashed lines refer to travel duration. *The equilibrium (black star) happens when total costs are equal between the two transport modes, for* $x_{e,publ} = x^*_{e,publ} = 0.11$.

For example, it is possible to analyze the impact of ticket fare on the mode choice of vehicles, which is the concern of this Section. Naturally, for a high enough fare, all vehicles will prefer the private transport mode, while for a low enough fare, all vehicles will choose the public one.

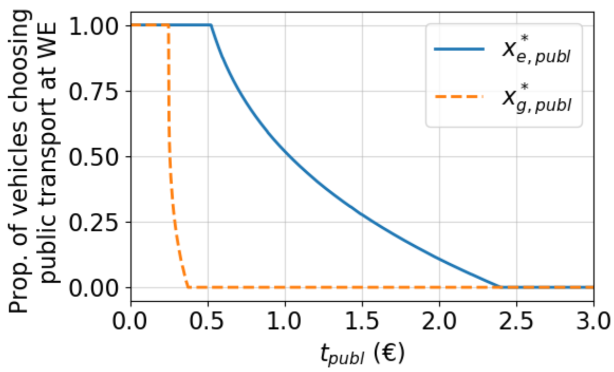
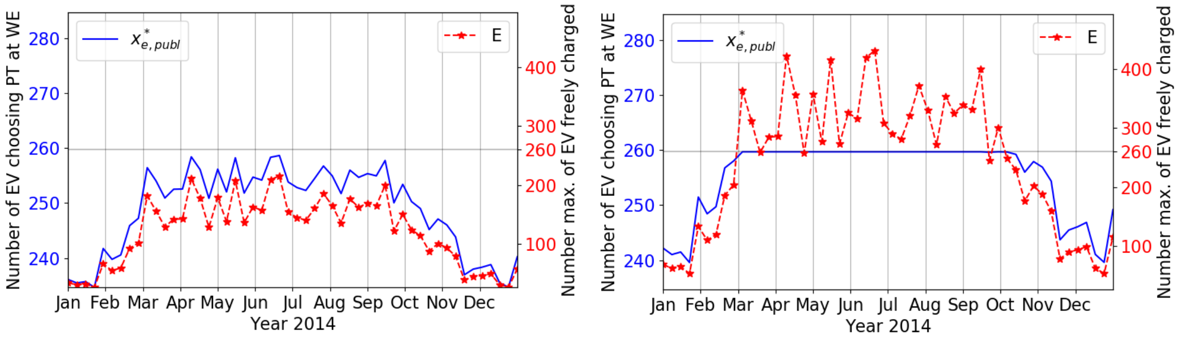


Figure 8: Proportion of EV and GV (dashed line) choosing the public transport mode at WE as a function of ticket fare. *For* $t_{publ} = 2.50€$ *(or greater), all vehicles choose the private mode; for* $t_{publ} = 0.25€$ *(or smaller), all vehicles choose the public one; for intermediate values EV are switching before GV due to attractiveness of unit charging price at the hub.*

Figure 8 shows the proportions of vehicles choosing the public transport mode in function of ticket fare, with the same parameters as in previous Section except that here there is enough PV production so that the hub charging service is free. Starting from the right side of the Figure and decreasing ticket fare from $t_{publ} = 3€$, EV are the first and only ones choosing the public transport mode instead of the private one when the ticket fare reaches $t_{publ} = 2.40€$. This is because EV have more to gain than GV in terms of consumption costs by switching from private to public mode, due to the large amount of PV production available. Some GV will choose the public mode only when all EV will have already chosen the public mode (around $t_{publ} = 0.50€$). No vehicle will be left downtown if the ticket fare is under $t_{publ} = 0.25€$.

3.3 PV panel sizing

For a fixed public transport ticket fare, the proportion of vehicles choosing the public transport mode might vary due to sunshine dependent PV production. For example, during the summer or in sunny locations, PV production is likely to be higher so that the charging unit price λ_e at the hub might be cheaper, thus inciting EV to choose the public transport mode rather than the private one. This is confirmed by the data from [19]: Figure 9 shows the evolution throughout the year 2014 of the number of EV choosing the public transport mode as a function of the PV production at the hub during this year. More precisely, the total number of vehicles is 1 000 (so that there are 500 EV) and for Figure 9b, a PV panel surface of 9 456m^2 is considered (so that $E = L_e^{\max}/2$, as the star curve in Figure 6a). For Figure 9b, a PV panel surface of 9 456/2 = 4 728m^2 was used. The parameters are the same as in last Sections, except for the ticket fare, set to $t_{publ} = 1$€. This Figure shows both the number of EV choosing the public transport mode at WE (blue line) and the number of EV that could have been freely charged - because exclusively from PV production (dashed line).



(a) PV surface: 4 728m^2. *Around 250 EV choose public transport while only 150 EV can be charged freely (on average).*

(b) PV surface: 9 456m^2. *At most 260 EV choose public transport while up to 400 EV could be freely charged.*

Figure 9: Evolution of the number of EV at the hub at equilibrium ($x^*_{e,publ}$) throughout year 2014. Evolution of the daily PV production, expressed in number of EV freely charged (dashed line). PT if for public transport.

Note that the order of magnitude of these two numbers is different (different left and right vertical scales). Also note that some EV choose the public transport mode even if the charging operation is not free (in Figure 9a, around 250 EV choosing public transport while only 150 EV can be charged freely on average) and that some EV choose the private mode even if the charging operation at the hub would be free (in Figure 9b, there are 260 EV at most choosing the public mode while up to 400 EV could be freely charged). The threshold effect - with a maximal number of 260 EV charging at the hub - observed in Figure 9b is due to the fact that the expected gain in consumption cost switching from private to public mode (i.e., from a 20c€/kWh charging unit price to a free one) is lower than the expected loss in travel duration. Note that in this scenario, all GV choose the private transport mode throughout the whole year, because of the attractiveness of downtown route with a small congestion.

4 Conclusion

This work focuses on the following scenario: electric and gasoline vehicles (EV and GV) can either drive all the way from the suburbs to the city center, or stop at an e-Park & Ride hub and continue by public transport. At the hub, a Charge Point Operator (CPO) is in charge of the charging scheduling in presence of a local PV production. The vehicles' choices are predicted taking into account congestion effects both on the traffic and on the electrical grid. The latter is represented here as a quadratic cost depending on the net curve at the charging station. Then, predictions of drivers' reaction to various control parameters can be made. For example, using real data of PV production, the CPO can compute the PV surface which maximizes its profits by inciting EV to stop and charge at the hub. Similarly, the public transport operator can compute the optimal ticket fare, to attract vehicles at the hub and minimize congestion and local pollution in the city center. In a future work, each EV will have the possibility to choose its own charging need based on the State of Charge when arriving at the hub (instead of being fully charged). Also, the case when the charging station is located on a site where other non-controlled electricity consumptions could be considered. In this case, the local PV production has to be shared between the charging usage and the other ones.

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